

FRACTAL METHODS IN COMPUTER GRAPHICS

Assignments set 1

1. Implement two basic algorithms of generating fractals given by IFS:

- (a) deterministic algorithm (Algorithm 1),
- (b) random algorithm (Algorithm 2).

Algorithm 1: Deterministic method

Input: $w_1, w_2, \dots, w_N : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ – affine mappings; $B \subset \mathcal{H}(\mathbb{R}^2)$ – the starting set (e.g., point, triangle); M – the number of iterations.

Output: Attractor's approximation.

```
1  $A = \{B\}$ 
2 for  $k = 0, 1, \dots, M - 1$  do
3    $T = \emptyset$ 
4   for  $j = 1, 2, \dots, N$  do
5     for  $i = 0, 1, \dots, |A| - 1$  do
6        $S = w_j(A_i)$ 
7        $T = T \cup S$ 
8    $A = T$ 
9 Draw all sets from  $A$ 
```

Algorithm 2: Random method (chaos game)

Input: $w_1, w_2, \dots, w_N : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ – affine mappings; p_1, p_2, \dots, p_N – probabilities; $q \in \mathbb{R}^2$ – the starting point; $M > 20$ – the number of iterations.

Output: Attractor's approximation.

```
1  $A = \emptyset$ 
2 for  $j = 0, 1, \dots, M - 1$  do
3    $i =$  random number from  $\{1, 2, \dots, N\}$  drawn with the probability distribution
4      $\{p_1, p_2, \dots, p_N\}$ 
5    $q = w_i(q)$ 
6   if  $j > 20$  then
7      $A = A \cup q$ 
7 Draw all points from  $A$ 
```

Exemplary IFSs are in the *ifs_fractals.zip* archive. The data format in the files is the following. The first line contains the number $N \in \mathbb{N}$ of affine mappings that form the IFS. In the next N lines, we have the coefficients of the mappings:

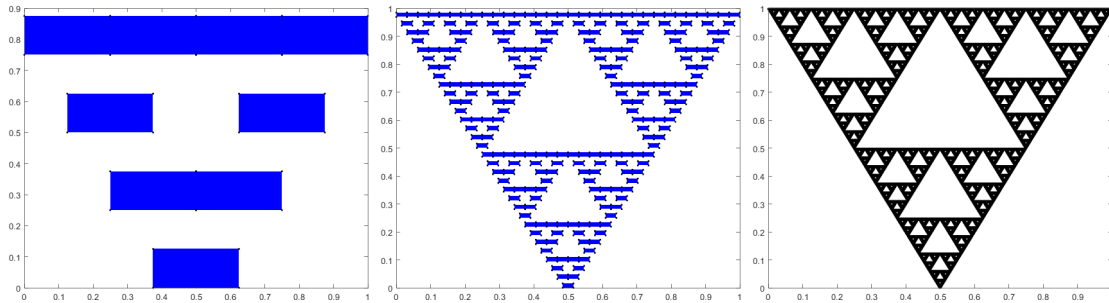
$a \ b \ c \ d \ e \ f \ p$

where $a, b, c, d, e, f \in \mathbb{R}$ are the coefficients of an affine mapping:

$$f([x, y]^T) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

and $p > 0$ is the probability.

Example of Sierpinski's triangle (*Sierpinski gasket 1.ifs*) generated by the deterministic method, starting from a quadrilateral. From left, iterations: 2, 5, 8.



Example of Barnsley's fern (*Barnsleys fern.ifs*) generated by the random method. From left, iterations: 10 000, 30 000, 100 000.

