

On Gumowski-Mira aesthetic superfractal forms

Wiesław Kotarski
Institute of Computer Science
University of Silesia
Sosnowiec, Poland
kotarski@math.us.edu.pl

Krzysztof Gdawiec
Institute of Computer Science
University of Silesia
Sosnowiec, Poland
kgdawiec@math.us.edu.pl

Agnieszka Lisowska
Institute of Computer Science
University of Silesia
Sosnowiec, Poland
alisow@math.us.edu.pl

Abstract—Gumowski-Mira transform, in short GM, produces nice looking fractal forms that can be used to model "marine living creatures" or aesthetic patterns useful for artistic design. Those original unrepeatable forms can inspire artistic design in jewellery such as pendants, necklaces, talismans. Moreover, GM can be a source of texture patterns for computer graphics and it suggests motives for fractal arts. We show that combination of GM with superfractals lead to enlarging a variety of fractal forms possible to create. Colours added to geometry enrich aesthetic appearance of superfractal forms generated with the help of GM.

Keywords-fractal; superfractal; IFS; Gumowski-Mira transformation; fractal art

I. INTRODUCTION

Fractals discovered by Mandelbrot in 1970s are used to model complex shapes such as clouds, plants, mountains, sea-shores. They are also applied in the field of art and design. Because of the possibility to create nice looking forms fractals may be a source of inspiration e.g. in producing 2D art images, animations, utility patterns and jewellery design. From ample literature devoted to fractals [1], [2], [6], fractal modelling and design [9], [10] we have chosen Gumowski-Mira transformation [3] due to its great potential to generate huge collection of fractal art forms [5], [7].

The next great idea – superfractals introduced by Barnsley in 2006 in his monograph [2] revealed new possibilities to create fractal forms that inherit shape features from fractal-parents. Thanks to superfractals one can obtain unlimited collection of fractal forms having potential value from the aesthetic point of view. Combination of GM transformations with superfractals, as we show in the paper, gives powerful possibilities in creation of new unrepeatable aesthetic forms.

II. GUMOWSKI-MIRA TRANSFORM

The Gumowski-Mira transform [7], in short GM transform, is a 2-dimensional nonlinear discrete dynamic system defined by the following recurrent formula:

$$\begin{aligned}x_{n+1} &= y_n + \alpha(1 - 0.05y_n^2)y_n + f(x_n), \\y_{n+1} &= -x_n + f(x_{n+1}), \\f(x) &= \mu x + \frac{2(1 - \mu)x^2}{1 + x^2},\end{aligned}\tag{1}$$

where x_0, y_0 is a starting point which can be chosen as an arbitrary point and $\alpha, \mu \in \mathbb{R}$ are the parameters of the GM transform. The dependency of the GM transform on two parameters α and μ we denote by $GM(\alpha, \mu)$.

This model has been introduced for modelling and study accelerated particles trajectories at CERN in 1980 [3]. Iterations of (1) produce different kind of cellular patterns reminding some kind of "living marine creatures" like a jellyfish, a starfish and a plankton. The output of GM transform is very sensitive to values of parameters α and μ . Changing those parameters one obtains a great variety of nice looking forms. In Fig. 1 some examples of patterns obtained using GM transform are presented. Detailed discussion and simulation results can be found in [7]. Especially interesting fractal forms can be obtained for parameters $\alpha = 0$ and μ equal to $-0.15, -0.31, -0.55, -0.75$ and very close to them. Generally speaking the values of parameters $\alpha = 0$ and $\mu \in (-0.95, -0.15)$ lead to nice fractal patterns.

III. FRACTALS AND SUPERFRACTALS

Fractals are used to describe complex graphical objects that cannot be represented with the help of classic Euclidean geometry. Fractals are fully described by the so-called IFS (Iterated Function System) and generated by deterministic or stochastic (Chaos game) algorithm. Sierpinski Gasket, Sierpinski Carpet, Koch curve, Koch snowflake and Barnsley's fern are well known examples of fractals. All of them are self-similar objects what means that a part of a fractal is similar to the whole fractal. The mathematical theory behind fractals is well described e.g. in [1], [6].

The main drawback of fractals is its ideal self-similarity that is rather rarely observed in Nature. So modelling natural objects with the help of classic fractals leads to non-realistic results. Take for example Barnsley's fern that can be obtained as a sum of many scaled, translated, rotated and sheared copies of itself. This leads to the situation in which all Barnsley's ferns are similar to each other in opposite to ferns met in reality. The real ferns are a little different from each other. That result can be obtained by the usage of some probabilistic mechanism which is implemented in superfractals.

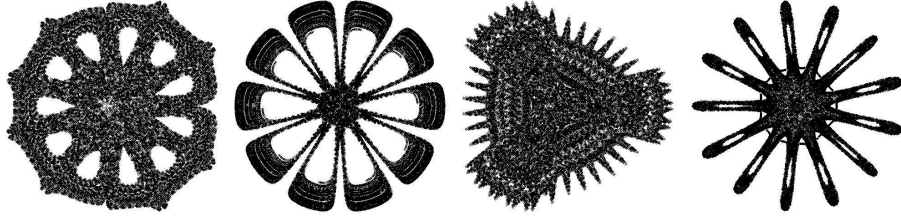


Figure 1. Gumowski-Mira fractals: $GM(0, -0.15)$, $GM(0, -0.31)$, $GM(0, -0.55)$, $GM(0, -0.75)$ from left to right, respectively. Starting point $[0, 0.5]^T$, number of iterations – 30000.

The notion of a superfractal can be explained on the example [1], [8] given below. Take two IFSs $F = \{f_1, f_2\}$ and $G = \{g_1, g_2\}$, where

$$\begin{aligned} f_1 \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{bmatrix} 1/2 & 3/8 \\ 1/2 & -3/8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1/16 \\ 9/16 \end{bmatrix}, \\ f_2 \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{bmatrix} 1/2 & -3/8 \\ -1/2 & -3/8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 9/16 \\ 17/16 \end{bmatrix}, \\ g_1 \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{bmatrix} 1/2 & 3/8 \\ -1/2 & 3/8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1/16 \\ 7/16 \end{bmatrix}, \\ g_2 \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{bmatrix} 1/2 & -3/8 \\ 1/2 & 3/8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 9/16 \\ -1/16 \end{bmatrix}. \end{aligned} \quad (2)$$

Attractors generated by the F and G IFSs take the form presented in Fig. 2, where the upper part of the fractal is generated by F , whereas the lower part by G .

Let us carry out the following experiment. In two buffers the left L and the right R arbitrary initial sets are put. Next, the contents of both buffers are modified using the following probabilistic mechanism [1]:

- 1) Select an IFS (F or G). Then select a buffer (L or R) and draw its content changed by the first transformation of the chosen IFS. Again, select a buffer (it may be the one used earlier) and draw it changed by the second transformation of the chosen IFS. Finally, sum the results (union of sets) of both transformations obtaining a new buffer L' .
- 2) Select an IFS (it may be the one used earlier). Then select a buffer (again L or R) and draw its content changed by the first transformation of the chosen IFS. Select another buffer and draw it changed by the second transformation of the chosen IFS. Finally, sum the results of both transformations obtaining a new buffer R' .
- 3) Let $L := L'$ and $R := R'$. Go to step 1.

After performing these steps new contents of buffers L and R are obtained. That process is repeated in such a way that probabilities of choosing the IFS and buffer are equal to $1/2$. After several iterations the obtained content of both buffers is independent on the initial sets, stored in both buffers at the beginning. Obtained attractors are probabilistic variations of the input objects – from fractal-parents are generated fractal-children that inherit some shape features

after fractal-parents. They form a collection of fractal forms called superfractal. Some elements of superfractal collection, resembling necklaces, are presented in Fig. 3. More examples of superfractals are presented e.g. in [4].

The experiment described above can be performed with larger number of IFSs and buffers. Also the number of transformations in the IFSs can be different and probabilities of choosing IFSs and buffers can be arbitrary. Because of probabilistic character of superfractal algorithm in every run of the program one can obtain different fascinating fractal forms. Observe, that in superfractal algorithm rather sets, not single points are transformed.

GM transform presents a single nonlinear transformation. So its IFS consists of only one transformation. Taking that into account now we can suggest the following superfractal algorithm for GM transformations:

Given data: starting point $[x_0, y_0]^T$, n number of iterations, Gumowski-Mira transformations with probabilities $GM(\alpha_1, \mu_1, p_1)$, $GM(\alpha_2, \mu_2, p_2)$ such that $p_1, p_2 > 0$, $p_1 + p_2 = 1$.

Assume that the k -th step of the algorithm is performed. Next it should be done:

- 1) Choose randomly one of the transforms: $GM(\alpha_1, \mu_1, p_1)$ or $GM(\alpha_2, \mu_2, p_2)$ and denote it by GM .
- 2) Transform the point $[x_k, y_k]^T$ by GM obtaining the point $[x_{k+1}, y_{k+1}]^T$. If the number of performed iterations is less than n go to step 1, else STOP and display the sequence of points $[x_k, y_k]^T$, $k = 0, \dots, n$.

The above algorithm can be easily generalized to the case with any finite number of GM transformations with probabilities.

IV. EXAMPLES

In Figs. 4 and 5 Gumowski-Mira fractal-parents and some superfractal-children obtained with the help of superfractal algorithm performing with probability $1/2$ after 30000 iterations, are presented. In both examples only four fractal children have been chosen from huge superfractal collection. It is easily seen that obtained superfractal-children inherit shape features after fractal-parents. It is so, if parameters of GM transforms have close values. In this situation one obtains stable fractal collection with nice looking fractal

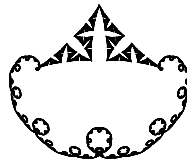


Figure 2. Two attractors (upper F and lower G).

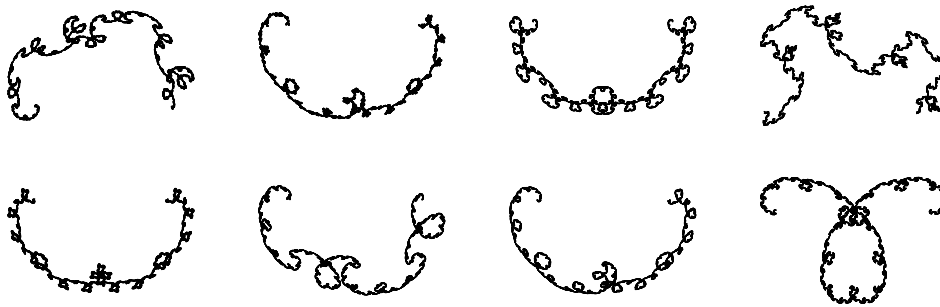


Figure 3. Some elements of superfractal collection obtained using F and G attractors.

forms. If differences between the values of parameters are large then the obtained fractal forms are not interesting from the aesthetic point of view. So, such cases are not presented here.

V. CONCLUSIONS

We showed that superfractal algorithm applied to Gumowski-Mira transformations produces collections of fractal forms having a nice looking geometrical appearance. Their geometry can be enriched by adding colours. Points of fractals can be easily coloured by a colour depending e.g. on their distance from a chosen point of fractal (centre of its mass), as in Fig. 6 or using other rules as in [5]. But it should be interesting to investigate how colours are inherited from GM fractal-parents by GM superfractal-children. Also it seems to be possible to apply Barnsley's steeling colours algorithm [1], [2] to GM superfractals.

The appearance of generated superfractal forms is unpredictable so not every form is potentially interesting from the artistic point of view. A man should choose forms from large superfractal collection those he likes. But there is also possible to evaluate artistic value of the generated superfractal forms automatically. It can be done using an approach basing on genetic algorithm presented e.g. in [9] for 2D fractals or in [10] for 3D case. It should be mentioned that in [2] Barnsley discusses relations between coefficients of IFS and DNA defining rules of creative systems. So applying genetic algorithms it is possible to control evolution creation process to obtain the forms one likes much.

Similar results, to those reported in the paper, can be obtained for other types of fractals e.g. Julia or Mandelbrot

sets. The mentioned above problems the authors of the paper would like to investigate in the future.

REFERENCES

- [1] M. F. Barnsley, "Superfractals", Cambridge University Press, New York, Melbourne, 2006.
- [2] M. F. Barnsley, "The life and survival of mathematical ideas", Notices of the AMS, vol. 57, no. 1, pp. 10-22, January 2010.
- [3] I. Gumowski and C. Mira, "Recurrences and Discrete Dynamic Systems", Springer 1980.
- [4] W. Kotarski and A. Lisowska, "Superfractal forms", <http://www.maplesoft.com/applications/view.aspx?SID=35009>.
- [5] H. B. Maallem, P. Richard, J-L. Ferrier and A. Labib, "Using Gumowski-Mira Maps for Artistic Creation", 12th Generative Art Conference GA2009.
- [6] B. Mandelbrot, "The Fractal Geometry of Nature", Freeman and Company, San Francisco, 1983.
- [7] K. Otsubo, M. Washida, T. Itoh, K. Katsuura and M. Hayashi, "Computer Simulation on the Gumowski-Mira Transformation", Forma, vol. 15, pp. 121-126, 2000.
- [8] S. L. Singh, S. Jain and S. N. Mishra, "A new approach to superfractals", Chaos, Solitons and Fractals, vol. 42, pp. 3110-3120, 2009.
- [9] S. Wannarumon and E. L. J. Bohez, "A New Aesthetic Evolutionary Approach for Jewelry Design", Computer-Aided Design & Applications, vol. 3, Nos. 1-4, pp. 385-394, 2006.
- [10] P. Wenjun and K. C. Hui, "Interactive Evolutionary 3D Fractal Modeling with Modified IFS", Computer-Aided Design & Applications, 6(1), pp. 55-67, 2009 DOI: 10.3722/cadaps.2009.55-67

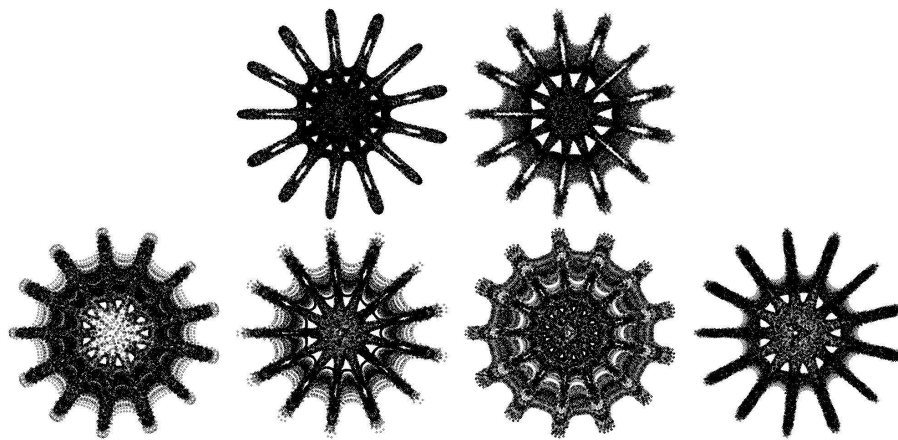


Figure 4. Up: fractal-parents; $GM(0, -0.7501, 0.5)$ and $GM(0, -0.7509, 0.5)$. Down: GM superfractal-children.

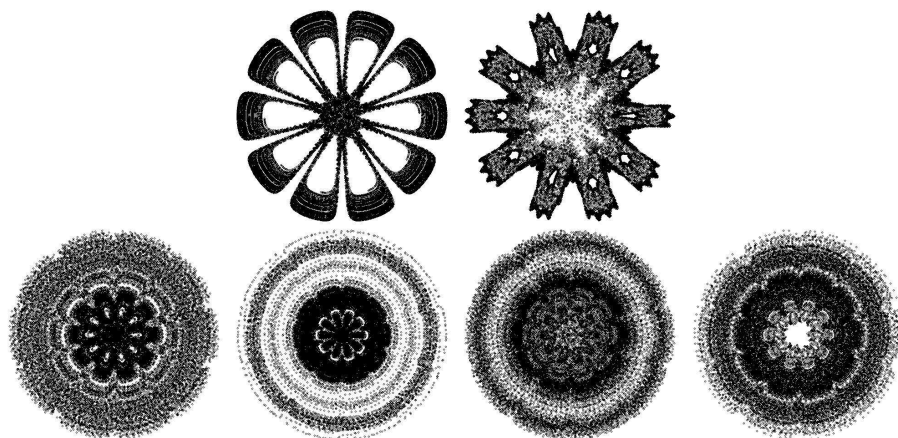


Figure 5. Up: fractal-parents; $GM(0, -0.31, 0.5)$ and $GM(0, -0.32, 0.5)$. Down: GM superfractal-children.

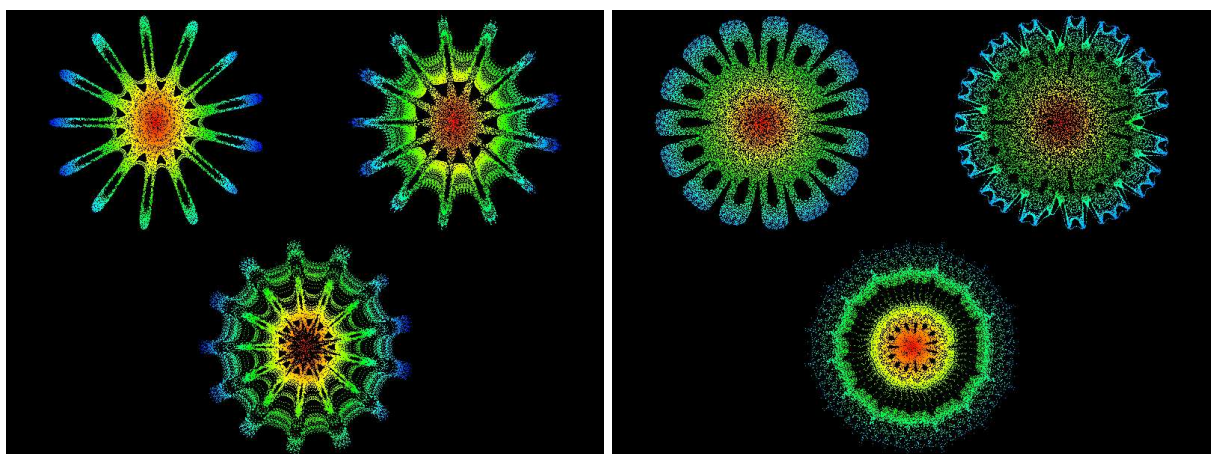


Figure 6. Up: coloured fractal-parents $GM(0, -0.7501, 0.5)$, $GM(0, -0.7509, 0.5)$ (for left), $GM(0, -0, 11, 0.5)$, $GM(0, -0.115, 0.5)$ (for right). Down: coloured GM superfractal-children.